Numerical Simulation of the Growth of Instabilities in Supersonic Free Shear Layers

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The behavior of the initial region of a supersonic plane shear layer is analyzed through numerical solution of the two-dimensional Navier-Stokes equations, as well as the three-dimensional equations under the finite-span assumption. A modified MacCormack scheme that is fourth-order accurate in space and second-order in time is employed. Small amplitude oscillations in the normal velocity are found to grow as they convect downstream, and eventually lead to organized vortical structures. Normal velocity disturbances are found to be more efficient than streamwise or spanwise disturbances. The growth rate of these disturbances, as well as the intensity of velocity fluctuations, are found to decrease as the convective Mach number of the shear layer increases. The Mach number of the vortical structures with respect to the faster stream is found to be considerably less than the theoretical value of the convective Mach number.

Nomenclature

a_1	= upper stream speed of sound
a_2	= lower stream speed of sound
F,G	= flux vectors
M_{vortex}	= convective Mach number of present result
M_c	= convective Mach number
q	= vector of conserved variables
R,S	= diffusion vectors
U_c	= convection speed of vortices
U_1	= upper stream inflow velocity
U_2	= lower stream inflow velocity
x	= coordinate in streamwise direction
y	= coordinate in normal direction

Introduction

AIR-BREATHING engines designed for high-flight Mach numbers require supersonic combustion for efficient operation. The shock losses associated with deceleration to low Mach numbers require that the mixing of fuel and air, and the heat release, must occur in supersonic flows. For the same reason, it is desirable to mix the fuel and air using coflowing streams. In such configurations, the mixing must occur across the shear layer formed between the streams. The length and weight of the engine, and the efficiency of heat release, depend on the rapidity of this mixing process. Most current concepts for supersonic-combustion ramjets thus employ mixing-limited heat release. The mixing across a shear layer between two streams depends on the rate of mass and momentum transfer across the layer, and, hence, can be described using the "growth" or "spreading" rate of the shear layer. Unfortunately, shear layers separating supersonic streams are known to grow much more slowly than corresponding subsonic shear layers.

One long-term objective of supersonic shear layer research, therefore, is to devise methods of increasing the mixing between supersonic streams by enhancing the shear layer growth rate. Recent success in greatly modifying subsonic shear layers has resulted in the advancement of a variety of schemes for achieving similar increases in supersonic shear layers. The variety of such possibilities far exceeds the resources available for experimental exploration of each. Instead, a better approach appears to be to develop reliable numerical models and solution methods that can then be used to perform the exploration, and to identify promising approaches and the appropriate values of parameters required. This is the motivation behind the research described in this paper.

Previous Work

Chinzei et al.[1] conducted experiments on planar shear layer configurations and studied the growth rate. Papamoschou[2] conducted similar experiments, using a variety of gases and flow conditions, and showed that the results could be scaled using the convective Mach number of the dominant eddies in the shear layer. These results showed that the growth rate of supersonic shear layers is typically less than one-third the growth rate of incompressible shear layers for convective Mach numbers greater than unity.

Passive and active control techniques have been studied by other researchers. These techniques are generally based on the principle that if vorticity is introduced into the shear layer, it will increase the level of fluctuation and, therefore, promote mixing and growth. Guirguis[3] and Drummond and Mukunda[4] studied the effect of a bluff body placed in the middle of the shear layer. Kumar et al.[5] considered the effects of vorticity produced by a pulsating shock wave on the growth characteristics of the shear layer. Ragab and Wu[6] have developed calculations based on stability theory to predict the response of supersonic shear layers. Recently,[7] they have also developed computations of the response of planar wakes and shear layers similar to those in experimental splitter plate configurations.

Scope of Present Paper

In the work presented here, the behavior of a planar free shear layer is studied, using two numerical techniques for solution of the Navier-Stokes equations. The effects of active

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control strategies are investigated. Sinusoidal variations in the velocity are introduced at the upstream boundary. The subsequent response of the shear layer to these disturbances is studied. Streamwise, normal, and spanwise disturbances are considered as suitable candidates for promoting mixing.

At present, the problem is assumed to be nominally two-dimensional. Some calculations have been performed with three-dimensional layers under the infinite sweep assumption. It is recognized that the later development of the shear layer may be strongly influenced by three-dimensional effects. However, there is no reason to believe that the initial region should be anything other than two-dimensional. The available experimental flow visualizations, performed with spanwise-integrating techniques such as schlieren and shadowgraphy, clearly show structures that would have been totally smeared out if the flowfields had been significantly three-dimensional.

The present calculations are for laminar shear layers, and no turbulence model is used. Turbulence models inherently bring additional uncertainty into the physical interpretation of the observed behavior of the flowfield, though they are certainly necessary to obtain quantitative accuracy. The lack of such a model restricts the applicability of these results to the initial region of the shear layer.

The initial velocity profile used is a step change in velocity at the slip line between the two streams. Thus, the results obtained will not correspond to experimental results from splitter-plate configurations, since there is no boundary layer and no embedded region of initially subsonic flow.

Within the above limitations, the present work aims to study the behavior of the initial region of a shear layer, and to explore the effects of various forms of excitation.

Problem Statement

The shear layer configuration is shown in Fig. 1. Two uniform, parallel supersonic streams of different Mach numbers are released at the left-hand boundary. All properties are known at this boundary. The upper and lower boundaries of the computational domain are assumed to be hard walls across which no disturbances can escape. There is no boundary layer at these walls, and slip conditions are used. At the downstream boundary, the flow and all disturbances are allowed to escape, and no disturbances are allowed to propagate back.

To study shear layer behavior, the static pressures are equalized across the splitter plate, so that there are no strong shocks in the flow. Some waves and their reflections from the wall do occur, but these are quite weak.

The flow is assumed to be nonreacting, and the ratio of specific heats was assumed to be constant for both streams. The species above and below the shear layer were assumed to have the same molecular weight.

Mathematical Formulation: Fourth Order MacCormack Scheme

$$q_t + F_x + G_y = R_x + S_y \tag{1}$$

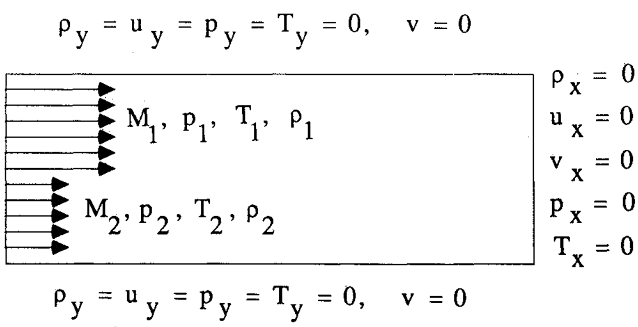

Fig. 1 Boundary conditions for supersonic free shear layer.

Here, F and G are the inviscid flux terms and account for the transport of mass, moment, and energy and for the influence of pressure. The terms R and S are the viscous stress terms. The above equation is parabolic with respect to time, and may be solved using a variety of stable time marching schemes. For two-dimensional flows, there are four equations. In the case of three-dimensional flows subject to infinite-sweep assumption, there are five equations, the additional equation corresponding to the conservation of spanwise momentum.

In this work, the above equation was solved using a splitting approach; that is, the solution was advanced from one time level 'n' to the next '$n+2$', through the following sequence of operations:

$$q^{n+2} = (L_x L_y L_{xv} L_{yv} L_{yv} L_{xv} L_y L_x) q^n \tag{2}$$

where, for example, the L_x operator involves solution of the following equation:

$$q_t + F_x = 0 \tag{3}$$

This one-dimensional equation was solved through the following predictor-corrector sequence, recommended by Bayliss et al.[8]:

Predictor Step:

$$q_{i,j^*} = q_{i,j^n} - \frac{\Delta t}{6\Delta x}\left[7F_{i,j} - 8F_{i-1j} + F_{i-2j}\right] \tag{4}$$

Corrector Step:

$$q_{i,j}{}^{n+1} = \frac{1}{2}\left(q_{i,j}{}^{n} + q_{i,j^*}\right) + \frac{\Delta t}{12\Delta x}\left[7F_{i,j} - 8F_{i+1,j} + F_{i+2,j}\right]^* \tag{5}$$

When the above equations are applied at nodes close to the left- and right-side boundary, a fourth-order accurate extrapolation procedure was used to extrapolate the flux vectors F and F^* needed at nodes outside the computational domain.

The L_y operator requires solution of the equation

$$q_t + G_y = 0 \tag{6}$$

using a similar approach.

The operators L_{xv} and L_{yv} correspond to numerical solution of one-dimensional equations such as

$$q_t - R_x = 0 \tag{7}$$

The above equation was solved through the following two-step sequence:

$$q_{i,j}{}^* = q_{i,j}{}^n + \frac{\Delta t}{\Delta x}\left[R_{i+\frac{1}{2},j} - R_{i-\frac{1}{2},j}\right]^n \tag{8}$$

$$q_{i,j}{}^{n+1} + \frac{1}{2}\left(q_{i,j}{}^n + q_{i,j}{}^*\right) + \frac{\Delta t}{2\Delta x}\left[R_{i+\frac{1}{2},j} - R_{i-\frac{1}{2},j}\right]^* \tag{9}$$

The viscous terms are thus updated only to second-order accuracy in space. It may be shown that the above scheme has very little artificial dissipation inherent in it, and is fourth-order accurate in space, as far as the inviscid part is concerned.

Boundary Conditions

As stated above, all flow properties are prescribed at the upstream boundary for both streams, including any imposed perturbations. At the downstream boundary, the flow is assumed to remain fully supersonic for the small-amplitude perturbations encountered in this work, so that the properties may be extrapolated from the interior. Alternatively, the governing equations themselves may be applied if the streamwise diffusion terms R_x are suppressed at the downstream nodes.

At the lateral boundaries, the flow is assumed to be confined by smooth, parallel walls. Slip boundary conditions were used to avoid the compression effects that would be caused by boundary layers. The walls were considered adiabatic, and the normal derivatives of density and pressure were set to zero.

Results and Discussion

Normalization

Velocities were normalized using the speed of sound in the upper stream, which thus became unity. The Reynolds number based on the speed of sound was chosen to be 1000. A 221×61 uniform grid was used, with spacing of 0.01 in each direction. Thus, the length of the domain L was 2.2. The time step was taken as 0.001 (0.0005 for each half-step). The calculations were started with step velocity profiles at the upstream boundary, and allowed to proceed until a steady state was reached asymptotically. This usually took 600 time steps. The results at this stage were stored, and the code was restarted with an imposed sinusoidal velocity disturbance of amplitude 2% of the velocity of the upper stream. The calculations were then run until several cycles of the disturbance had been completed, and the initial effects had been convected away through the downstream boundary.

Convective Mach Number

The cases run have been summarized in Table 1. Because the supersonic shear flow problem involves several parameters (at least five on either side of the shear layer), nondimensional groupings are sought to express observed effects. Following the practice of Papamoschou,[2] the convective Mach number was used here. For the problem studied here, the convective Mach number reduces to

$$M_c = (U_1 - U_c)/a_1$$

where

$$U_c = (a_1 U_2 + a_2 U_1)/(a_1 + a_2)$$

The values of M_c calculated by this formula are tabulated. A physical interpretation of the convective Mach number is that it is the Mach number of the dominant large-scale vortical structures with respect to either stream. According to the formula given above, this Mach number is the same with respect to either of the streams. An attempt was made, as discussed later, to determine the convection speed of the vortical structures seen in the computational flowfield, and to determine relative Mach numbers from them. The Mach numbers so determined, with respect to the upper, high-speed stream, are also tabulated. It is seen that there is a considerable discrepancy. This is not surprising, and, in fact, even in subsequent experiments by Papamoschou,[9] similar effects appear to have been observed.

Table 1 Cases presented

Case	M_1	M_2	U_1	U_2	M_c	M_{vortex}
1	4.0	2.3	4.00	3.51	0.20	0.2
2	4.0	2.0	4.00	3.05	0.38	0.2
3	4.0	1.3	4.00	1.98	0.80	0.2
4	5.0	1.3	5.00	1.98	1.20	0.6

Formation of Vortical Structures

Figure 2a shows the contours of vorticity in the shear layer, calculated for case 1, with no disturbance superposed. It is seen that the shear layer grows quickly at the very beginning, and then takes on a smooth profile which grows very little thereafter. It should be remembered that in this calculation there is no imposed turbulence model. Figure 2b shows the effect of imposing a sinusoidal 2% normal velocity disturbance at the inflow boundary. Distinct centers of vorticity are seen to develop and be convected downstream. The shear layer edge now penetrates considerably further into both streams. Careful examination of the contours shows considerable asymmetry and distortion as the structures proceed downstream. The computational domain in this calculation does not extend far enough for these disturbances to grow into the nonlinear regime, and hence no "roll-up" can be expected here. The effects of six cycles of the imposed disturbance can be seen, with the sixth just leaving the computational domain.

Figures 3a and 3b show the corresponding vorticity contours for case 2, where the theoretical value of convective Mach number is nearly twice that of case 1. The growth rate

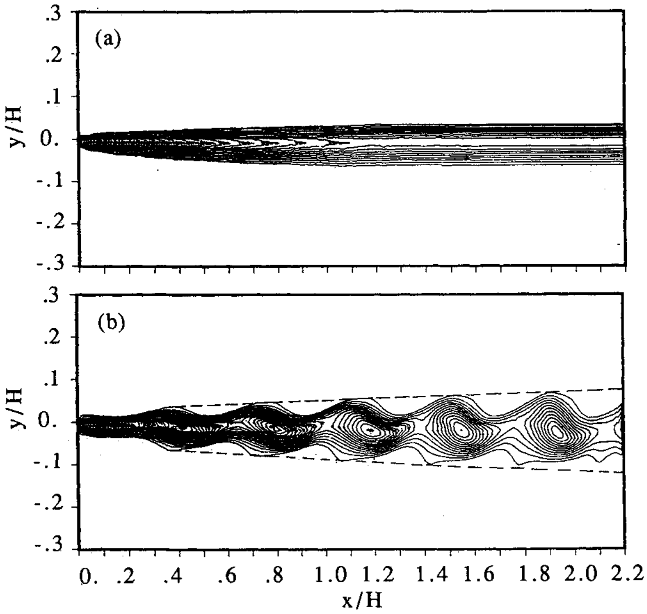

Fig. 2 Vorticity contours for case 1, $M_1 = 4.0$, $M_2 = 2.3$, $M_c = 0.2$.; a) without disturbance, b) with disturbance in the normal direction.

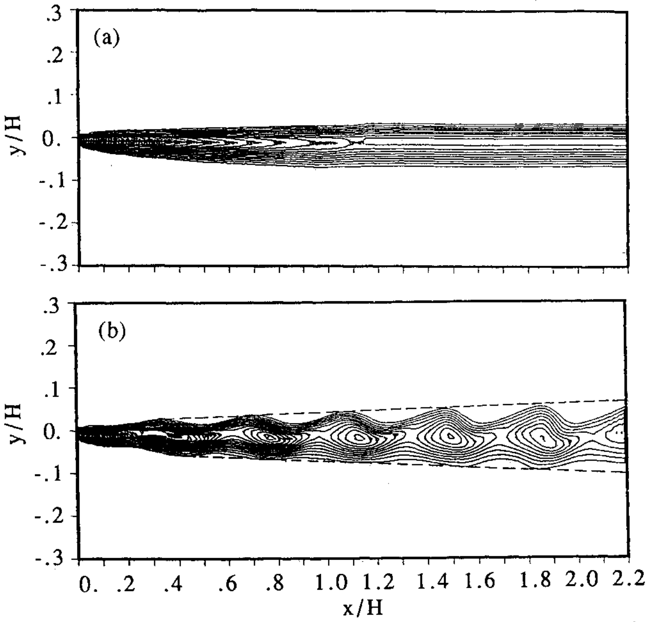

Fig. 3 Vorticity contours for case 2, $M_1 = 4.0$, $M_2 = 2.0$, $M_c = 0.38$; a) without disturbance, b) with disturbance in the normal direction.

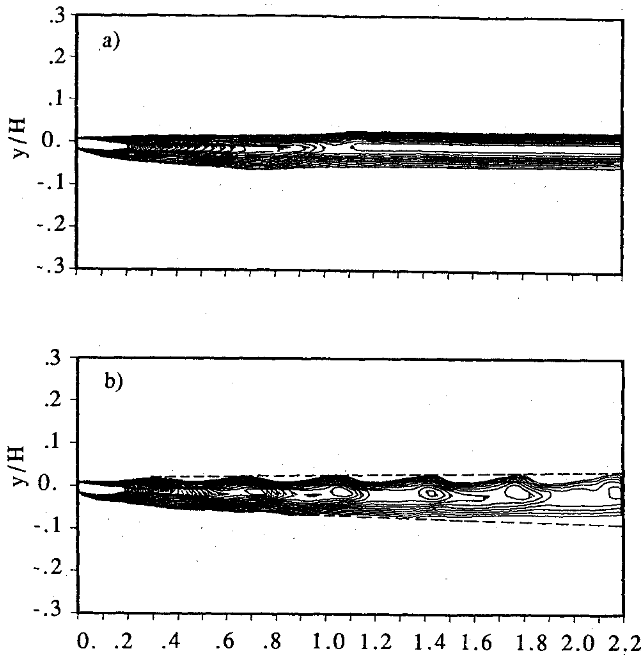

Fig. 4 Vorticity contours for case 3, $M_1 = 4.0$, $M_2 = 1.3$, $M_c = 0.8$; a) without disturbance, b) with disturbance in the normal direction.

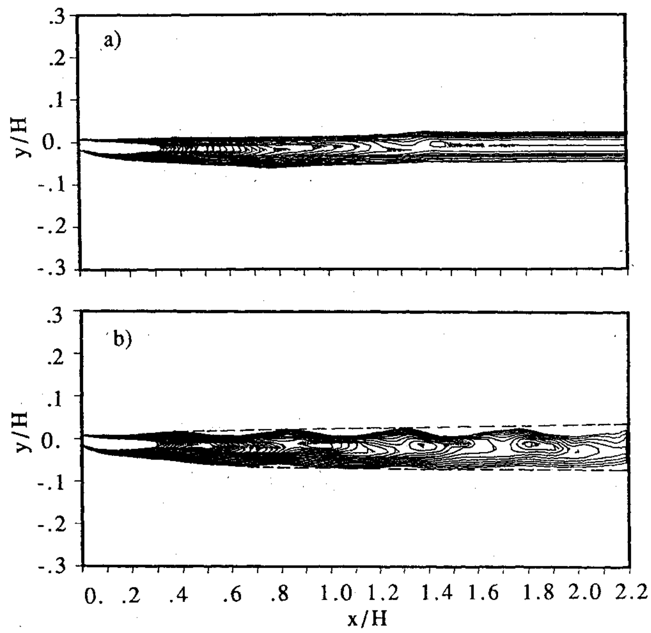

Fig. 5 Vorticity contours for case 4, $M_1 = 5.0$, $M_2 = 1.3$, $M_c = 1.2$; a) without disturbance, b) with disturbance in the normal direction.

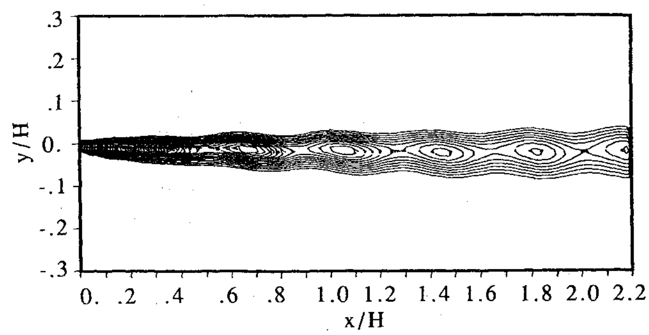

Fig. 6 Effect of streamwise disturbances for case 1.

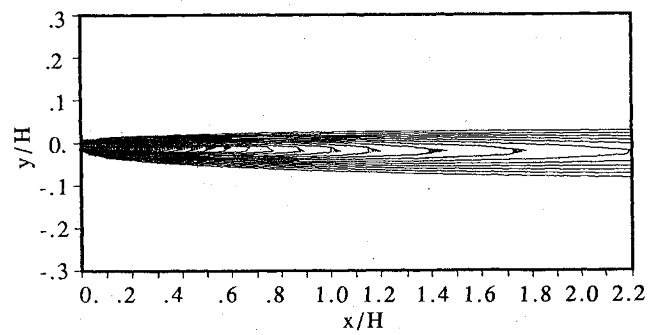

Fig. 7 Effect of spanwise disturbances for case 1.

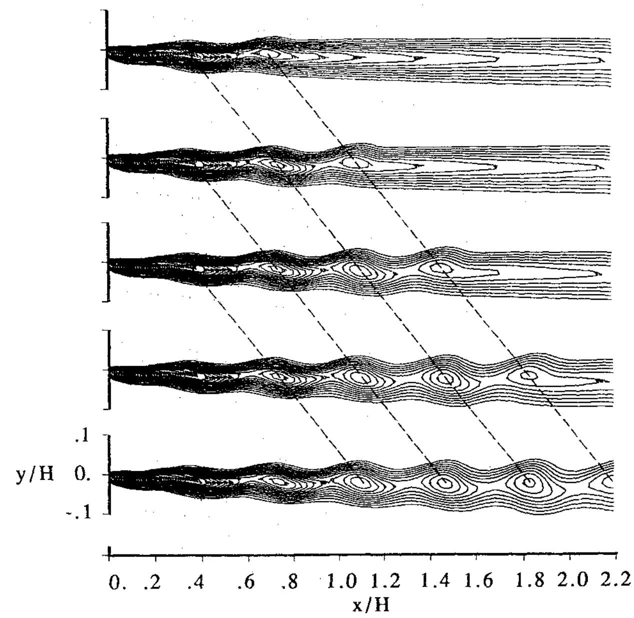

Fig. 8 Computation of convective speed based on the temporal evolution of vorticity contours for case 2.

appears to be less, as expected from the experimental observations of the effect of M_c. This effect is seen further in Figs. 4 and 5, where the convective Mach number, according to the formula, is 0.8 and 1.2, respectively.

Disturbance Type

Other kinds of disturbance were tried; specifically, disturbances in velocity along the streamwise direction for the two-dimensional shear layer and disturbances along the spanwise direction using a three-dimensional model with the infinite sweep assumption. The results are shown in Figs. 6 and 7, respectively, for the case where the convective Mach number is predicted to be 0.2. In each case, the disturbance amplitude is the same. It is seen that these types of disturbances are less efficient than the normal velocity disturbance.

Convective Speed Based on Evolution of Vorticity Contours

One common way of calculating the convective Mach number is to track the downstream convection of the vorticity contours as a function of time. The results of this effort are shown in Fig. 8 for case 2. Similar computations were performed for all of the cases, and the results are shown in Table 1. It is seen that as the theoretical value of the convective Mach number rises, the measured convective Mach number of the structures with respect to the upper stream vary much less. Of course, this means that the convective Mach number is no longer the same when compared to the two streams. The structures appear to move at a speed that is close to that of the upper, high-speed stream. This is similar to the more recent observations of Papamoschou,[9] where the structures were found, for many cases, to move at speeds close to that of one or the other stream.

Growth Rate Enhancement

The thickness of the shear layer was computed from the velocity profiles across the shear layer. These profiles are shown later in the paper, where they are used to examine the numerical accuracy of the results. Figure 9 shows the shear layer thickness for unperturbed and perturbed cases for case 1. The disturbed case shows a significantly greater rate of growth, except near the downstream boundary. However, the increase in growth rate is only on the order of 10-15%.

The frequency of the imposed fluctuations was chosen such that about six vortical structures could be seen in the computational domain at one time. Thus, the actual frequency used was higher than the frequency of maximum amplification predicted by linear stability analysis. Use of the preferred frequency would have required a much larger computational domain in the x direction to capture an adequate number of vortices, and would have increased the computational effort required by an order of magnitude.

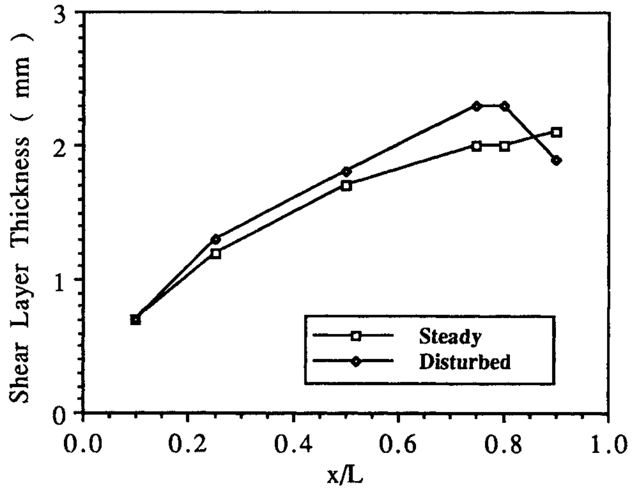

Fig. 9 Shear layer thickness.

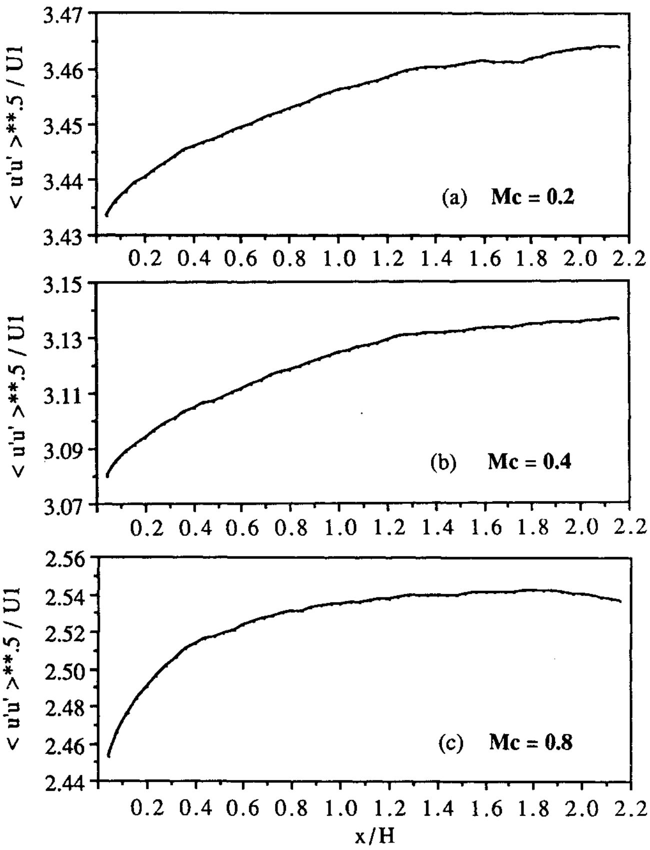

Fig. 10 Effect of convective Mach number on fluctuation in the shear layer; a) $M_c = 0.2$; b) $M_c = 0.38$; and c) $M_c = 0.80$.

The intensity of fluctuation in the shear layer is plotted as a function of downstream distance for three cases with different theoretical convective Mach numbers in Figs. 10a-10c. The quantity measured was the root-mean-squared fluctuation of the U-component of velocity about the mean, normalized by the mean velocity. It is to be noted that this is not the turbulence intensity, since no attempt has been made to model the turbulence. The fluctuations have their origin in the imposed disturbance, though they may have been selectively amplified by energy exchange with the shear layer. It is seen that the intensity of fluctuations decreases rapidly with increasing convective Mach number. It also appears that the intensity quickly reaches an asymptotic value and does not increase further. In fact, for the higher values of convective Mach number, the intensity appears to peak and then decrease gradually thereafter. This decay in the intensity with high values of M_c has been predicted by other researchers using linear stability analysis.

Studies of Discretization Error

All the above results were generated using the fourth-order MacCormack scheme. The influence of the accuracy of the computation scheme was studied by comparing results obtained using a second-order MacCormack scheme to those obtained with the fourth-order scheme. Velocity profiles across the shear layer were used to examine the results. Figure 11 shows the comparison for case 1, however, with a 111×31 grid. The profile was obtained at the station 10% of the domain length downstream of the origin. The results are seen to be quite similar. The agreement is close for profiles at 25 and 50% downstream. However, at $x/L = 0.75$, differences can be clearly seen. The fourth-order scheme is seen to resolve spatial details better, as expected. The difference decreased

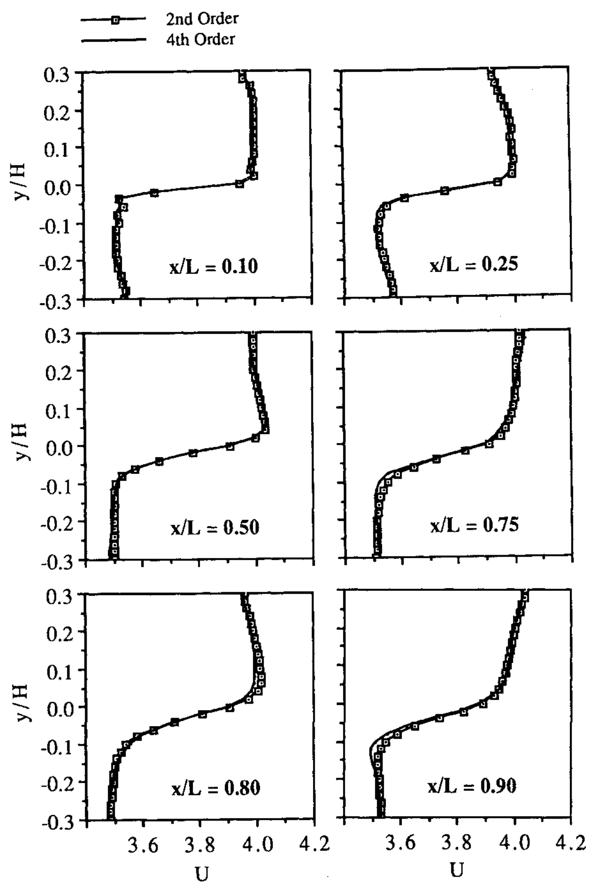

Fig. 11 Velocity profiles across the shear layer: comparison of second- and fourth-order MacCormack schemes for case 1 with 111×31 grid.

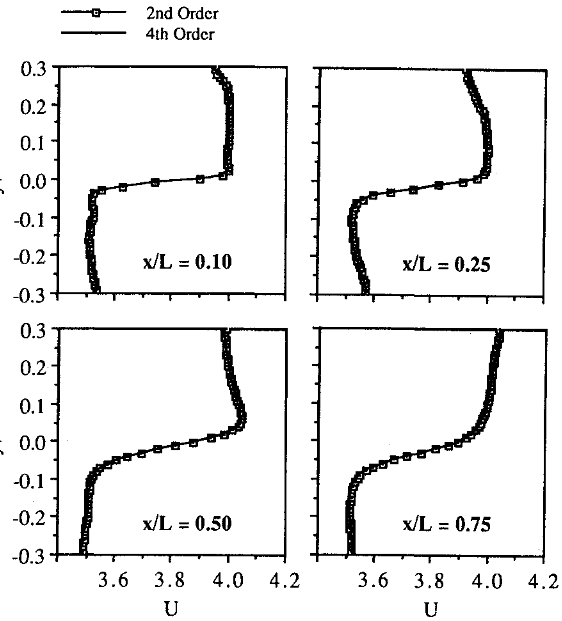

Fig. 12 Velocity profiles across the shear layer: comparison of second- and fourth-order MacCormack schemes for case 1 with 221 × 61 grid.

further downstream. It is concluded from these that the second-order scheme already shows reasonable accuracy, and that the fourth-order results are quite accurate.

The effect of grid size was checked by comparing the above results with those obtained with the 221 × 61 grid, as shown in Fig. 12. In this case, the difference between the second- and fourth-order codes is negligible for all stations. Thus, it is concluded that with the 221 × 61 grid, the results are not sensitive to grid size.

Computational Resources

All calculations reported here were performed on the Cray X-MP at the Pittsburgh Supercomputing Center. The CPU time per time step per grid node was 10 μs for the fourth-order MacCormack scheme, using the 221 × 61 grid.

Conclusions

A numerical study of the behavior of planar supersonic shear layers has been performed. Different numerical schemes have been tested. Techniques for enhancing the growth rate of the shear layer have been investigated. The following are noted from the results:

1) The fourth-order MacCormack scheme accurately simulates the evolution of the imposed disturbances. The results are essentially independent of the spatial order of the scheme, with a 221 × 61 grid with the grid parameters used.

2) A perturbation of 2% of the mean flow velocity, imposed at the upstream boundary in the normal direction, produces a larger growth of the shear layer than an equal amount of perturbation imposed in the streamwise or spanwise directions.

3) Imposed sinusoidal disturbances in the normal velocity upstream lead to the formation and growth of vortical structures. The shear layer thickness grows rapidly at first and then the growth rate decreases asymptotically.

4) The root-mean-square fluctuation level in the streamwise velocity and the shear layer growth rate decrease with increasing values of the theoretical convective Mach number of the shear layer.

5) The vortical structures are found to move at different Mach numbers relative to the upper and lower stream, and the relative Mach number appears to be smaller relative to the stream with the higher Mach number.

Acknowledgments

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